Engineering Notes

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Optimization of Payload Placement on Arbitrary Spacecraft

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Nomenclature

$G_j\ (X)$	= inequality constraint equation j
$H_k\ (X)$	= equality constraint equation k
I	= inertia matrix
I_{xx}, I_{yy}, I_{zz}	= inertia moments
$J(x,y,z)$	= objective function (performance index)
U_v^2	= unit vector, pointing toward nadir, in a spacecraft-centered, Earth-oriented coordinate system
U_v^3	= unit vector, along the vertical, from the Earth's center to the spacecraft
γ_i	= weighting factor for term i in performance index
θ	= pitch angle off spacecraft local vertical
τ_g	= gravity gradient torque vector
$\tau_{gx}, \tau_{gy}, \tau_{gz}$	= gravity gradient torque vector components
ω	= Earth orbital rate

Introduction

STUDYING the Earth from orbiting spacecraft requires collecting many types of measurements recorded over long periods of time. Each measurement is usually made by a specialized instrument and thus has requirements for viewing, thermal control, vibrations, and power. To accommodate these instruments, the spacecraft must provide the required resources. Since the host spacecraft has a finite load capability, several goals must be achieved. For example, instrument capabilities on the spacecraft should be maximized, and spacecraft bus consumables should be minimized over the mission lifetime. The goal of minimizing bus consumables can become rather involved. One of the primary resources to be conserved is the attitude control and stationkeeping propellant. To this end, the spacecraft must be balanced such that the principal axes of inertia coincide with the spacecraft's viewing requirements. In this case, the propellant needed would be for disturbance correction.

The only way to accomplish such balancing on a specified spacecraft configuration is to relocate external payloads about the spacecraft. Each payload has mass and inertia, thus providing means to alter the overall mass properties on the spacecraft. The performance of the instruments must not be compromised at the cost of degrading the performance of the spacecraft. Many instruments have specific pointing requirements, as well as electromagnetic interference (EMI) criteria that cannot be ignored. Also, physical constraints must be considered.

A systematic method for determining an optimal placement of instrumentation on arbitrary spacecraft is presented. The method maximizes resource utilization by minimizing the spacecraft's need for propulsive attitude control. A nonlinear mathematical program will be presented along with considerations toward reducing the size of the optimization effort. A representative payload set will be used as an example, and the results will be discussed.

Gravity Gradient Torque Equations

As stated earlier, minimizing the cross products of inertia will also minimize the gravity gradient torques experienced in orbit. The gravity gradient torques on an Earth-oriented body are governed by the following matrix equation[1]:

$$\tau_g \equiv 3\omega^2 U_v^3 \times [I] U_v^{3T} \tag{1}$$

The coordinate system used is a normal Cartesian coordinate system fixed at the body's center of mass with the x axis aligned with the direction of flight, the y axis perpendicular to the orbital plane (pointing toward the North Pole), and the z axis pointing toward nadir. A set of Euler angles can be employed to reorient the body to another position from this initial alignment.

The unit vector U_v^3 is a function of the spacecraft's Euler angles. It is the vector dot product of the Euler-angle rotation matrix[2] $[E]$ and a unit vector directed towards nadir in a spacecraft-centered, Earth-oriented coordinate system:

$$U_v^3 = [E] U_v^2 \tag{2}$$

The torque components are shown in their expanded forms below:

$$\tau_{gx} = 3\omega^2 \begin{bmatrix} \tfrac{1}{2}\sin(2\phi)(I_{33} - I_{22})\cos^2\theta + \tfrac{1}{2}\sin(2\theta)\,(I_{21}\cos\phi - I_{31}\sin\phi) \\ - I_{32}\cos^2\theta\cos(2\theta) \end{bmatrix} \tag{3}$$

$$\tau_{gy} = 3\omega^2 \begin{bmatrix} [\tfrac{1}{2}\sin(2\theta)(I_{33} - I_{11})\cos\phi + I_{32}\sin\phi] + \tfrac{1}{2}\sin(2\phi)(I_{12}\cos^2\theta) \\ + I_{13}(\cos^2\phi\cos^2\theta - \sin^2\theta) \end{bmatrix} \tag{4}$$

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$$\tau_{gz} = 3\omega^2 \begin{bmatrix} \tfrac{1}{2}\sin(2\theta)[(I_{11} - I_{22})\sin\phi - I_{23}\cos\phi] + I_{12}(\sin^2\theta - \cos^2\theta\sin^2\phi) \\ - I_{13}\cos^2\theta(\tfrac{1}{2}\sin 2\phi) \end{bmatrix} \tag{5}$$

The above equations indicate that the gravity gradient torques are a function of the Euler angles ϕ and Θ. The Euler angle Θ, in the presence of zero ϕ and ψ Euler angles, is also the angle that the spacecraft's nadir-pointing axis could be pitched from the local vertical. If one investigates the dependency of the pitch angle Θ on the torques, the major effects on the pointing of the spacecraft are those about the x axis and the y axis. In examining these components of the gravity gradient torque vector, the sine term is "small," and the cosine term is near unity for small values of Θ. Therefore, in the absence of large differences in the inertia moments, the primary driver of the torque components is the inertia cross product. Thus, by minimizing the cross-product terms, there will be a minimal contribution to the torque components.

Mathematical Program Formulation

Mathematical programming techniques require a performance index to be minimized (maximized) as well as a set of constraint equations under which the minimization is to take place. A program would be derived to solve the problem:

Minimize:

$$J(\bar{X}) \tag{6}$$

Subject to:

$$G_j(\bar{X}) \leq \bar{0}, \qquad j = 1,\ldots,n \tag{7}$$

$$H_k(\bar{X}) \leq \bar{0}, \qquad k = 1,\ldots,m \tag{8}$$

This particular mass-properties optimization problem is easily conformed to a parameter optimization problem. In this case, a performance index is required so that the minimization of the inertia cross products are accommodated.[3] The performance index to be minimized becomes

$$J(x,y,z) = \gamma_1 I_{xy}^2 + \gamma_2 I_{xz}^2 + \gamma_3 I_{yz}^2 \tag{9}$$

This type of problem is indifferent to non-positive cross-product values; that is, if the sum of the products were used as the performance index, a large negative cross-product term can cancel a large positive cross-product term and the sum of the terms can be near zero or a high negative number in the minimization. Large negative values are not desirable in this optimization. It is then necessary to force the cross-product terms to be non-negative, thus justifying the square terms in the performance index. An additional variable γ_i that functions as a weighting factor is included in the performance index so that tailoring of the results to fit the designer's requirements could be achieved. By allowing γ_i to reside in the set $(0,\infty)$, the inertia products could be weighted to reflect the desires of the analyst.

This performance index is subject to the following constraints: each payload must stay on the spacecraft, each payload must not occupy the same space of any other, and if applicable, payloads will not cause electromagnetic interference with other payloads. As an example of a set of physical constraints, consider three payload elements placed side-by-side. These three elements generate four physical constraints. Two constraints disallow the end elements from being located off of the spacecraft along the x axis. The other two constraints keep the center element from occupying the regions outlined by the two end elements. The addition of side constraints accommodates the physical limitation in the y and z axes. This defines the feasible region for a solution. Mission constraints are used in similar fashion as physical constraints. Often the placement of a particular payload at a particular location on the spacecraft is desirable. Those constraints would be applied as a set of equality constraints. The EMI constraints are used to further restrict the movement of payloads and to not interfere or degrade the performance of another payload. The source of the electromagnetic energy of a payload is assumed to reside at its geometric center. The constraint equation defines a sphere with a radius equal to the minimum distance that any other payload can approach this particular payload as defined by the EMI restriction. An example would be if an instrument has an EMI constraint of 3 m, then the closest any other instrument can approach would be on the surface of a 3-m-diam sphere centered at the geometric center of the instrument with the constraint. All of these considerations are integrated into a nonlinear mathematical program for optimization.

Example Problem

The optimization methodology has been used as the basis for the development of an automated computer program. This program optimizes the placement of instrument sets on a spacecraft, while adhering to specified constraints. The program is built upon a general parameter optimization code created by Vanderplaats.[4] The code, Automated Design Synthesis (ADS), solves a general class of optimization problems based upon linear and/or nonlinear objective functions and constraint sets. The following example will present an application of the methodology to a specific spacecraft concept.

Table 1 Positions of payloads before and after optimation

Payload	Initial positions			Final positions		
	X	Y	Z	X	Y	Z
1	−8.000	4.500	3.000	−7.884	1.369	3.000
2	4.000	3.000	3.000	3.810	2.620	3.000
3	7.000	2.000	3.000	6.588	2.300	3.000
4	9.000	−2.000	3.000	7.677	−1.252	3.000
5	−9.000	2.000	−3.000	−7.649	1.128	−3.000
6	7.000	−4.000	−3.000	7.025	−1.220	−3.000
7	−5.000	5.500	2.000	−4.164	5.500	1.356
8	5.000	5.500	−1.000	6.775	5.500	−2.000
9	−2.000	−5.500	−1.000	3.053	−5.500	−0.269
10	4.000	−5.500	−2.000	−3.419	−5.500	2.000

Table 2 Inertia matrices for example case

$$I_{initial} = \begin{bmatrix} 29,290 & & \\ -8940 & 64,210 & \\ 1780 & 4903 & 78,660 \end{bmatrix} \qquad I_{final} = \begin{bmatrix} 25,210 & & \\ 0.0023 & 60,120 & \\ 0.3659 & -0.0322 & 70,960 \end{bmatrix}$$

The spacecraft bus is a solid rectangular structure 20 m long (length in x direction), 10 m wide (width in y direction), and 5 m deep (depth in z direction). The spacecraft has a mass of 1000 kg and the following inertia properties: I_{xx} = 5208 kg-m^2, I_{yy} = 17,703 kg-m^2, I_{zz} = 20,833 kg-m^2. The spacecraft inertia cross products are 0 due to symmetry about the body axes. In this example, the payload set consists of a group of 1-m cubes, each weighing 100 kg, uniformly distributed about the cube's volume and placed at arbitrary positions on the spacecraft. Each cube's inertia is also accounted for. The example case consists of a 10-instrument set distributed as follows: four on the positive z face of the spacecraft, two on the negative z face of the spacecraft, two on the positive y face, and two on the negative y face. In this case, the weighting factors γ_i were set to 1.0, and thus equal importance is placed on each inertia product.

The spacecraft and the instruments are represented as parallelepipeds. The program accommodates a complete mass-properties description of each item in terms of center-of-gravity placement and inertia properties.

After the optimization was performed, the results show a reduction of the inertia cross products by an average of four orders of magnitude. The initial and final positions of the instruments are shown in Table 1. The initial and final inertia matrices results are in Table 2. The gravity gradient torque magnitude was reduced from 1.95×10^{-2} to 1.37×10^{-6} N-m for a 250 n.mi. orbit. Based on these results, the amount of propellant budgeted for gravity gradient-induced momentum desaturation can be reduced, and additional instruments can be added.

Obviously, a simplistic rectangular structure may not be available for spacecraft optimization. Through the use of additional constraints, such as side constraints and equality/inequality constraints, the problem can be sufficiently tailored to a specific application. The inertia cross products are significantly reduced using this methodology. The objective function could be altered to account for other concerns, such as center-of-gravity location and weight, volume, or power requirements.

Optimization techniques can be used to determine the position of externally attached payloads on any arbitrary spacecraft, thus providing insight on the effect of mass-properties management on a spacecraft system. Given a set of payloads on any face of the spacecraft, a mathematical model can be created as point masses with individual inertia matrices, and optimization can be accomplished using available parameter optimization programs. Using an example spacecraft concept, this approach demonstrated that payload locations can be altered to reduce the inertia cross products. In this particular analysis, the inertia products were reduced an average four orders of magnitude. In the development of further mission constraints, many mass-properties analyses can be performed in a quick and efficient manner.

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Large Solar Proton Events and Geosynchronous Communication Spacecraft Solar Arrays

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Introduction

THE design levels of power margins in geosynchronous solar arrays and of weight penalties by solar array cover glasses depend not only on the magnetosphere electron fluxes but also on the solar flare proton fluence models used. The occurrence of the high fluence solar events in October 1989 has caused a re-examination of some of the assumptions made as to the appropriate energy spectral representations of the large solar flare proton events used for some engineering designs. We discuss several aspects of solar flare proton fluxes and conclude that exponential in rigidity spectral representations of the largest events in the last two decades should be used for design purposes.

Solar Proton Fluences

Charged-particle instrumentation flown on the first geosynchronous spacecraft, ATS-1, returned data that demonstrated the relatively ready access of solar flare particles to the Earth's magnetosphere.[1] Because of the easy access of solar flare particles,[2,3] it was readily evident that these particles had to be included in determinations of the radiation dosage expected for synchronous spacecraft components. In particular, the easy access of relatively low-energy solar particles ($\lesssim$ 10 MeV, and even $\lesssim$1 MeV) meant that these particles could be significant factors in producing damage to solar arrays, damage that could significantly influence the margins required in the power design.[4-6]

The relatively benign interplanetary conditions, in terms of high intensities of solar flare particle fluxes, that persisted for years following the August 1972 solar event and throughout

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